Eur. Phys. J. C 9, 335–342 (1999)
DOI 10.1007/s100529900060

THE EUROPEAN PHYSICAL JOURNAL C

Universal seesaw mass-matrix model and SO(10)×SO(10) unification

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Received: 28 December 1998 / Published online: 20 May 1999

Abstract. In the universal seesaw mass-matrix model, which is a promising model for the unified description of the quark and lepton mass matrices, the behaviors of the gauge coupling constants and intermediate energy scales in the $\mathrm{SO}(10)_\mathrm{L} \times \mathrm{SO}(10)_\mathrm{R}$ model are investigated in relation to the neutrino-mass generation scenarios. The unification of the gauge coupling constants in the framework of the non-SUSY model is possible if the SO(10) symmetry is broken via Pati–Salam-type symmetries.

1 Introduction

Recently, considerable interest [1–5] in the universal seesaw mass-matrix model [6] has revived, this being a unified mass-matrix model of the quarks and leptons. First suggested by the seesaw mechanism for neutrinos [7], the model was then proposed in order to understand the question why the masses of quarks (except for the top quark) and charged leptons are so small compared with the electroweak scale Λ_L ($\sim 10^2$ GeV). The model has hypothetical fermions F_i in addition to the conventional quarks and leptons f_i (flavors $f = u, d, \nu, e$; family indices $i = 1, 2, 3$), and they are assigned to $f_\mathrm{L} = (2, 1)$, $f_\mathrm{R} = (1, 2)$, $F_\mathrm{L} = (1, 1)$ and $F_\mathrm{R} = (1, 1)$ of $\mathrm{SU}(2)_\mathrm{L}\times\ \mathrm{SU}(2)_\mathrm{R}$. The 6 × 6 mass matrix which is sandwiched between the fields $(\overline{f}_\mathrm{L}, \overline{F}_\mathrm{L})$ and $(f_\mathrm{R}, F_\mathrm{R})$ is given by

$$M^{6\times6} = \begin{pmatrix} 0 & m_\mathrm{L} \\ m_\mathrm{R} & M_F \end{pmatrix}, \tag{1.1}$$

where m_L and m_R are universal for all fermion sectors ($f = u, d, \nu, e$) and only the M_F have structures dependent on the flavors f. For $\Lambda_\mathrm{L} < \Lambda_\mathrm{R} \ll \Lambda_\mathrm{S}$, where $\Lambda_\mathrm{L} = O(m_\mathrm{L}), \Lambda_\mathrm{R} = O(m_\mathrm{R})$ and $\Lambda_\mathrm{S} = O(M_F)$, the 3 × 3 mass matrix M_f for the fermions f is given by the well-known seesaw expression

$$M_f \simeq -m_\mathrm{L} M_F^{-1} m_\mathrm{R}. \tag{1.2}$$

However, after the observation [8] of the heavy top quark mass $m_t \sim \Lambda_\mathrm{L}$, the model at one stroke became embarrassed, because the observed fact $m_t \sim O(m_\mathrm{L})$ means $O(M_F^{-1} m_\mathrm{R}) \sim 1$. This problem was recently solved by Fusaoka and the author [1], and later by Morozumi et al. [2]. If we can build a model with $\det M_F = 0$ for the up-quark sector ($F = U$), one of the fermion masses $m(U_i)$ is zero [say, $m(U_3) = 0$], so that the seesaw mechanism does not work for the third family, i.e., the fermions $(u_{3\mathrm{L}}, U_{3\mathrm{R}})$ and $(u_{3\mathrm{R}}, U_{3\mathrm{L}})$ acquire masses of order $O(m_\mathrm{L})$ and $O(m_\mathrm{R})$, respectively. We identify $(u_{3\mathrm{L}}, U_{3_\mathrm{R}})$ as the top quark $(t_\mathrm{L}, t_\mathrm{R})$. Thus, we can understand the question why only the top quark has a mass of the order of Λ_L. Of course, we can successfully describe [1] the quark masses and mixings in terms of the charged-lepton masses by assuming simple structures for $m_\mathrm{L}, m_\mathrm{R}$ and M_F. The model also gives an interesting phenomenology for neutrinos [3].

In spite of such phenomenological success, there is reluctance to accept the model, because the model needs extra fermions F. In most unification models, there is no room for the fermions F. For example, it has been found [4] that when the gauge symmetries $\mathrm{SU}(2)_\mathrm{L}\times\mathrm{SU}(2)_\mathrm{R}\times \mathrm{U}(1)_Y\times\mathrm{SU}(3)_\mathrm{c}$ are embedded into the Pati–Salam-type [9] unification $\mathrm{SO}(10) \to \mathrm{SU}(2)_\mathrm{L}\times\mathrm{SU}(2)_\mathrm{R}\times\mathrm{SU}(4)_\mathrm{PS}$, those gauge coupling constants are unified at $\mu = \Lambda_\mathrm{X} \simeq 6 \times 10^{17}$ GeV [SU(4) is broken into $\mathrm{U}(1)_Y\times\mathrm{SU}(3)_\mathrm{c}$ at $\mu = \Lambda_\mathrm{R} \simeq 5 \times 10^{12}$ GeV]. However, in the SO(10) model there is no representation which offers suitable entries for the fermions $F_{\mathrm{L/R}} = (1, 1, 4)_{\mathrm{L/R}}$ of $\mathrm{SU}(2)_\mathrm{L}\times\mathrm{SU}(2)_\mathrm{R}\times\mathrm{SU}(4)_\mathrm{PS}$. Whether we can built a unification model in which the fermions F are reasonably embedded will be a touchstone for the future of the universal seesaw mass-matrix model.

There is an idea which might offer a solution to this problem [10]. We may consider the fermions F_R^c ($\equiv C\overline{F}_\mathrm{R}^\mathrm{T}$) together with the fermions f_L to belong to **16** of SO(10), and also F_L^c together with f_R to belong to **16** of another SO(10), i.e.,

$$(f_\mathrm{L} + F_\mathrm{R}^c) \sim (16, 1), \quad (f_\mathrm{R} + F_\mathrm{L}^c) \sim (1, 16), \tag{1.3}$$

of $\mathrm{SO}(10)_\mathrm{L}\times\mathrm{SO}(10)_\mathrm{R}$. The symmetries are broken into $\mathrm{SU}(2)_\mathrm{L}\times\mathrm{SU}(2)_\mathrm{R}\times\mathrm{U}(1)_Y\times\mathrm{SU}(3)_\mathrm{c}$ at $\mu = \Lambda_\mathrm{S}$ and the fermions F have a mass term $\overline{F}_\mathrm{L} M_F F_\mathrm{R}$.

In order to examine the idea of (1.3), in the present paper we investigate the evolution of the gauge coupling

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constants on the basis of the $\mathrm{SO}(10)_\mathrm{L}\times\mathrm{SO}(10)_\mathrm{R}$ model and estimate the intermediate energy scales Λ_R and Λ_S together with the unification energy scale Λ_X. As regards the numerical results, we are particularly interested in the value of $\kappa \equiv \Lambda_\mathrm{R}/\Lambda_\mathrm{L}$, because this value is closely related to the neutrino-mass generation scenarios, as we discuss in the next section. The evolutions of the gauge coupling constants under $\mathrm{SO}(10)_\mathrm{L}\times\mathrm{SO}(10)_\mathrm{R}$ symmetries have already been analyzed by Davidson, Wali and Cho [10], but their symmetry-breaking patterns are somewhat different from that in the present model. We will investigate the possible intermediate energy scales under the constraint $\Lambda_\mathrm{R}/\Lambda_\mathrm{S} \simeq 0.02$ [1] as derived from the observed ratio m_t/m_c in the new scenario of the universal seesaw model [1,2], where the masses m_t and m_c are given by $m_t \sim \Lambda_\mathrm{L}$ and $m_c \sim (\Lambda_\mathrm{R}/\Lambda_\mathrm{S})\Lambda_\mathrm{L}$, respectively.

In Sect. 3, we investigate the case of the symmetry breaking $\mathrm{SO}(10)_\mathrm{L}\times\mathrm{SO}(10)_\mathrm{R} \to [\mathrm{SU}(5)\times\mathrm{U}(1)']_\mathrm{L}\times[\mathrm{SU}(5)\times\mathrm{U}(1)']_\mathrm{R}$. We will see that we should rule out this case, because the results are inconsistent with the observed values of the gauge coupling constants at $\mu = m_Z$. In Sect. 4, we investigate the case $\mathrm{SO}(10)_\mathrm{L} \times \mathrm{SO}(10)_\mathrm{R} \to [\mathrm{SU}(2) \times \mathrm{SU}(2)' \times \mathrm{SU}(4)]_\mathrm{L} \times [\mathrm{SU}(2) \times \mathrm{SU}(2)' \times \mathrm{SU}(4)]_\mathrm{R}$. We will conclude that this case is allowed for the intermediate energy scale $\Lambda_\mathrm{R} \sim (10^1 - 10^6)$ GeV if we accept a model with $\Lambda_\mathrm{XL} \neq \Lambda_\mathrm{XR}$, where Λ_XL and Λ_XR are the unification scales of $\mathrm{SO}(10)_\mathrm{L}$ and $\mathrm{SO}(10)_\mathrm{R}$, respectively. Finally, Sect. 5 will be devoted to conclusions and final remarks.

2 Neutrino mass matrix

In the universal seesaw mass-matrix model, the most general form of the neutrino-mass matrix which is sandwiched between $(\overline{\nu}_\mathrm{L}, \overline{\nu}^c_\mathrm{R}, \overline{N}_\mathrm{L}, \overline{N}^c_\mathrm{R})$ and $(\nu^c_\mathrm{L}, \nu_\mathrm{R}, N^c_\mathrm{L}, N_\mathrm{R})^\mathrm{T}$ is given by

$$M^{12\times 12} = \begin{pmatrix} 0 & 0 & m'_\mathrm{L} & m_\mathrm{L} \\ 0 & 0 & m^\mathrm{T}_\mathrm{R} & m'^T_\mathrm{R} \\ m'^T_\mathrm{L} & m_\mathrm{R} & M_\mathrm{R} & M_\mathrm{D} \\ m^\mathrm{T}_\mathrm{L} & m'_\mathrm{R} & M^\mathrm{T}_\mathrm{D} & M_\mathrm{L} \end{pmatrix}, \tag{2.1}$$

under the broken $\mathrm{SU}(2)_\mathrm{L} \times SU(2)_\mathrm{R} \times \mathrm{U}(1)_Y$ symmetries. Here, we have denoted the Majorana mass terms of the fermions F^c_L and F^c_R as M_R and M_L, respectively, because the fermions F_L and F_R are members of $(1, 16^*)$ and $(16^*, 1)$ of $\mathrm{SO}(10)_\mathrm{L}\times\mathrm{SO}(10)_\mathrm{R}$, respectively. The mass terms $\overline{f}_\mathrm{L} m_\mathrm{L} F_\mathrm{R}$ and $\overline{F}_\mathrm{L} m_\mathrm{R} f_\mathrm{R}$ are generated, for example, by the Higgs scalars $(126, 1)$ and $(1, 126^*)$ of $\mathrm{SO}(10)_\mathrm{L} \times \mathrm{SO}(10)_\mathrm{R}$, respectively, while the mass terms $\overline{f}_\mathrm{L} m'_\mathrm{L} F^c_\mathrm{L}$ and $\overline{F}^c_\mathrm{R} m'_\mathrm{R} f_\mathrm{R}$ must be generated by Higgs scalars of the type $(16, 16^*)$ of $\mathrm{SO}(10)_\mathrm{L}\times\mathrm{SO}(10)_\mathrm{R}$. Therefore, in the present model we do not consider the terms m'_L and m'_R, i.e., we take $m'_\mathrm{L} = m'_\mathrm{R} = 0$. (For the special case with $m'_\mathrm{L} \simeq m_\mathrm{L}$ and $m'_\mathrm{R} \simeq m_\mathrm{R}$, see [11].) Hereafter, we assume $m_\mathrm{L} \ll m_\mathrm{R} \ll M_F$.

We are interested in a mass matrix for the left-handed neutrino states ν_L. By using the seesaw approximation for the matrix (2.1), we obtain the 6×6 mass matrix for the approximate $(\nu^c_\mathrm{L}, \nu_\mathrm{R})$ states,

$$\begin{aligned} M^{6\times 6} &\simeq -\begin{pmatrix} 0 & m_\mathrm{L} \\ m^\mathrm{T}_\mathrm{R} & 0 \end{pmatrix} \begin{pmatrix} M_\mathrm{R} & M_\mathrm{D} \\ M^\mathrm{T}_\mathrm{D} & M_\mathrm{L} \end{pmatrix}^{-1} \begin{pmatrix} 0 & m_\mathrm{R} \\ m^\mathrm{T}_\mathrm{L} & 0 \end{pmatrix} \\ &= -\begin{pmatrix} m_\mathrm{L} M^{-1}_{22} m^\mathrm{T}_\mathrm{L} & m_\mathrm{L} M^{-1}_{21} m_\mathrm{R} \\ m^\mathrm{T}_\mathrm{R} M^{-1}_{12} m^\mathrm{T}_\mathrm{L} & m^\mathrm{T}_\mathrm{R} M^{-1}_{11} m_\mathrm{R} \end{pmatrix}, \end{aligned} \tag{2.2}$$

where

$$\begin{pmatrix} M_\mathrm{R} & M_\mathrm{D} \\ M^\mathrm{T}_\mathrm{D} & M_\mathrm{L} \end{pmatrix}^{-1} = \begin{pmatrix} M^{-1}_{11} & M^{-1}_{12} \\ M^{-1}_{21} & M^{-1}_{22} \end{pmatrix} \tag{2.3}$$

$$\begin{aligned} M_{11} &= M_\mathrm{R} - M_\mathrm{D} M^{-1}_\mathrm{L} M^\mathrm{T}_\mathrm{D}, M_{22} = M_\mathrm{L} - M^\mathrm{T}_\mathrm{D} M^{-1}_\mathrm{R} M_\mathrm{D}, \\ M_{12} &= M^\mathrm{T}_{21} = M^\mathrm{T}_\mathrm{D} - M_\mathrm{L} M^{-1}_\mathrm{D} M_\mathrm{R}. \end{aligned} \tag{2.4}$$

Corresponding to the cases (a) $M_\mathrm{L}, M_\mathrm{R} \gg M_\mathrm{D}$, (b) $M_\mathrm{L}, M_\mathrm{R} \sim M_\mathrm{D}$ and (c) $M_\mathrm{L}, M_\mathrm{R} \ll M_\mathrm{D}$, we obtain the following mass matrix for the approximate ν_L states.

(a) The case $M_\mathrm{L}, M_\mathrm{R} \gg M_\mathrm{D}$

From $M_{11} \simeq M_\mathrm{R}$, $M_{22} \simeq M_\mathrm{L}$ and $M_{12} \simeq -M_\mathrm{L} M^{-1}_\mathrm{D} M_\mathrm{R}$, we obtain

$$M^{6\times 6} \simeq \begin{pmatrix} -m_\mathrm{L} M^{-1}_\mathrm{L} m^\mathrm{T}_\mathrm{L} & m_\mathrm{L} M^{-1}_\mathrm{L} M^\mathrm{T}_\mathrm{D} M^{-1}_\mathrm{R} m_\mathrm{R} \\ m^\mathrm{T}_\mathrm{R} M^{-1}_\mathrm{R} M_\mathrm{D} M^{-1}_\mathrm{L} m^\mathrm{T}_\mathrm{L} & -m^\mathrm{T}_\mathrm{R} M^{-1}_\mathrm{R} m_\mathrm{R} \end{pmatrix}, \tag{2.5}$$

so that we get the mass matrix for the approximate ν_L states,

$$M(\nu_\mathrm{L}) \simeq -m_\mathrm{L} M^{-1}_\mathrm{L} m^\mathrm{T}_\mathrm{L}, \tag{2.6}$$

because of $(M^{6\times 6})_{11}, (M^{6\times 6})_{22} \gg (M^{6\times 6})_{12}$.

(b) The case $M_\mathrm{L}, M_\mathrm{R} \sim M_\mathrm{D}$

We consider the case

$$\det \begin{pmatrix} M_\mathrm{R} & M_\mathrm{D} \\ M^\mathrm{T}_\mathrm{D} & M_\mathrm{L} \end{pmatrix} \neq 0. \tag{2.7}$$

(The special case that the determinant is zero has been discussed in [3].) Since we consider the case $m_\mathrm{L} \ll m_\mathrm{R}$, we can use the seesaw approximation for the expression (2.2), so that we obtain

$$\begin{aligned} M(\nu_\mathrm{L}) &\simeq -m_\mathrm{L} M^{-1}_{22} m^\mathrm{T}_\mathrm{L} \\ &\quad + m_\mathrm{L} M^{-1}_{21} m_\mathrm{R} (m^\mathrm{T}_\mathrm{R} M^{-1}_{11} m_\mathrm{R})^{-1} m^\mathrm{T}_\mathrm{R} M^{-1}_{12} m^\mathrm{T}_\mathrm{L} \\ &= -m_\mathrm{L} (M^{-1}_{22} - M^{-1}_{21} M_{11} M^{-1}_{12}) m^\mathrm{T}_\mathrm{L} \\ &= -m_\mathrm{L} M^{-1}_\mathrm{L} m^\mathrm{T}_\mathrm{L}, \end{aligned} \tag{2.8}$$

where we have used the relation $M_\mathrm{L} = (M^{-1}_{22} - M^{-1}_{21} M_{11} \cdot M^{-1}_{12})^{-1}$ in the inverse expression of (2.3). Thus, we obtain the expression (2.6) for the case (b), too. Note that the 3×3 mass matrix for the approximate ν_L states is almost independent of the structures of M_D and M_R in spite of $O(M_{\rm L}) \sim O(M_{\rm D}) \sim O(M_{\rm R})$.

(c) The case $M_{\rm L}, M_{\rm R} \ll M_{\rm D}$

From $M_{11} \simeq -M_{\rm D} M_{\rm L}^{-1} M_{\rm D}^{\rm T}$, $M_{22} \simeq -M_{\rm D}^{\rm T} M_{\rm R}^{-1} M_{\rm D}$ and $M_{12} \simeq M_{\rm D}^{\rm T}$, we obtain the mass matrix

$$M^{6\times 6} \simeq \begin{pmatrix} m_{\rm L} M_{\rm D}^{-1} M_{\rm R} M_{\rm D}^{T-1} m_{\rm L}^{\rm T} & -m_{\rm L} M_{\rm D}^{-1} m_{\rm R} \\ -m_{\rm R} M_{\rm D}^{T-1} m_{\rm L}^{\rm T} & m_{\rm R}^{\rm T} M_{\rm D}^{T-1} M_{\rm L} M_{\rm D}^{-1} m_{\rm R} \end{pmatrix}. \tag{2.9}$$

The mass matrix gives three light pseudo-Dirac neutrino states [12] $\nu_{i\pm}^{{\rm ps}D} \simeq (\nu_{i{\rm L}} \pm \nu_{i{\rm R}}^c)/\sqrt{2}$ $(i = e, \mu, \tau)$, because $(M^{6\times 6})_{11}, (M^{6\times 6})_{22} \ll (M^{6\times 6})_{12}$. This case has been discussed by Bowes and Volkas [13]. It is very attractive phenomenologically, because the maximal mixing state between $\nu_{\mu{\rm L}}$ and $\nu_{\mu{\rm R}}$ can give a natural explanation for the recent atmospheric neutrino data [15]. The mass matrix $M(\nu_\pm^{{\rm ps}D})$ in the limit of $m(\nu_{i+}^{{\rm ps}D}) = m(\nu_{i-}^{{\rm ps}D})$ is approximately given by

$$M(\nu_\pm^{{\rm ps}D}) \simeq -m_{\rm L} M_{\rm D}^{-1} m_{\rm R}. \tag{2.10}$$

First, we suppose the following symmetry-breaking pattern (hereafter, we will refer to this as case (A)):

$$\begin{array}{c} {\rm SO(10)_L \times SO(10)_R} \\ \downarrow \quad \mu = \Lambda_{\rm X10} \\ {\rm [SU(5) \times U(1)']_L \times [SU(5) \times U(1)']_R} \\ \downarrow \quad \mu = \Lambda_{\rm N} \\ {\rm SU(5)_L \times SU(5)_R} \\ \downarrow \quad \mu = \Lambda_{\rm X5} \\ {\rm [SU(3) \times SU(2) \times U(1)]_L \times [SU(3) \times SU(2) \times U(1)]_R} \\ \downarrow \quad \mu = \Lambda_S \\ {\rm SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)}_Y \\ \downarrow \quad \mu = \Lambda_{\rm R} \\ {\rm SU(3)_c \times SU(2)_L \times U(1)}_{Y'} \\ \downarrow \quad \mu = \Lambda_{\rm L} \\ {\rm SU(3)_c \times U(1)_{em}}. \end{array} \tag{2.11}$$

At the energy scale $\mu = \Lambda_{\rm N}$, the gauge symmetries ${\rm U(1)'}_{\rm L} \times {\rm U(1)'}_{\rm R}$ are completely broken, so that the neutral leptons $N_{\rm L}$ and $N_{\rm R}$ acquire Dirac and Majorana masses of the order of $\Lambda_{\rm N}$. At $\mu = \Lambda_{\rm S}$, the remaining fermions $F_{\rm L}$ and $F_{\rm R}$ (except for $U_{3{\rm L}}$ and $U_{3{\rm R}}$) acquire masses of the order of $\Lambda_{\rm S}$ by Higgs bosons Φ (as we discuss in the next section), and ${\rm SU(3)_L \times SU(3)_R}$ and ${\rm U(1)_L \times U(1)_R}$ are broken into ${\rm SU(3)_{L+R} \equiv SU(3)_c}$ and ${\rm U(1)_{L+R}} \equiv {\rm U(1)}_Y$, respectively. If scenario (A) is correct, the mass matrices $M_{\rm L}, M_{\rm R}$ and $M_{\rm D}$ are of the order of $\Lambda_{\rm N}$, so that we suppose that the order of the neutrino masses $m(\nu_i)$ is given by

$$m(\nu_i) \sim \Lambda_{\rm L}^2/\Lambda_{\rm N} \sim (\Lambda_{\rm L}\Lambda_{\rm S}/\Lambda_{\rm R}\Lambda_{\rm N}) m(e_i), \tag{2.12}$$

from the result (2.8) in case (b), the neutrino masses are suppressed by a factor $(\Lambda_{\rm L}/\Lambda_{\rm R})$ $(\Lambda_{\rm S}/\Lambda_{\rm N})$ as compared with the charged-lepton masses $m(e_i)$.

Next, we can suppose another symmetry breaking (case (B)):

$$\begin{array}{c} {\rm SO(10)_L \times SO(10)_R} \\ \downarrow \quad \mu = \Lambda_{\rm X} \\ {\rm [SU(2) \times SU(2)' \times SU(4)]_L \times [SU(2) \times SU(2)' \times SU(4)]_R} \\ \downarrow \quad \mu = \Lambda_S \\ {\rm SU(2)_L \times SU(2)_R \times U(1)}_Y {\rm \times SU(3)_c} \\ \downarrow \quad \mu = \Lambda_{\rm R} \\ {\rm SU(3)_c \times SU(2)_L \times U(1)}_{Y'} \\ \downarrow \quad \mu = \Lambda_{\rm L} \\ {\rm SU(3)_c \times U(1)_{em}}. \end{array} \tag{2.13}$$

If scenario (B) is true, since $M_{\rm L} \sim M_{\rm R} \sim M_{\rm D} \sim M_{\rm S}$, we suppose

$$m(\nu_i) \sim \Lambda_{\rm L}^2/\Lambda_{\rm S} \sim (\Lambda_{\rm L}/\Lambda_{\rm R}) m(e_i), \tag{2.14}$$

so that the neutrino masses $m(\nu_i)$ are suppressed by a factor $\Lambda_{\rm L}/\Lambda_{\rm R}$ as compared with the charged-lepton masses $m(e_i)$.

It is of great interest to estimate the possible values of such intermediate energy scales $\Lambda_{\rm R}$, $\Lambda_{\rm S}$, and so on.

Although the Bowes–Volkas model [13] is very interesting, this model cannot apply in the universal seesaw model based on the ${\rm SO(10)_L \times SO(10)_R}$ unification, because the case $M_{\rm L}, M_{\rm R} \ll M_{\rm D}$ is not likely in the ${\rm SO(10)_L \times SO(10)_R}$ model, and, if it would be adequate, the relation (2.10) leads to the wrong prediction $m(\nu_i) \sim m(e_i)$ for $M_{\rm D} \equiv M_{\rm N} \sim M_F$ $(F \neq N)$.

3 Case of $\mathbf{SO(10) \to SU(5) \times U(1)}$

In the present section, we investigate case (A) with the symmetry-breaking pattern (2.11). At the energy scale $\mu = \Lambda_{\rm S}$, the symmetries ${\rm [SU(3) \times SU(2) \times U(1)]_L \times [SU(3) \times} SU(2) \times U(1)]_{\rm R}$ are broken into ${\rm SU(3)_c \times SU(2)_L \times U(1)}_Y$ by the following Higgs scalars Φ_Y:

$$\Phi_{2/3} \sim (3^*, 1; 3, 1)_{Y=2/3}, \ \ \Phi_{4/3} \sim (3, 1; 3^*, 1)_{Y=4/3}, \tag{3.1}$$
$$\Phi_2 \sim (1, 1; 1, 1)_{Y=2},$$

of ${\rm [SU(3) \times SU(2)]_L \times [SU(3) \times SU(2)]_R}$, where ${\rm SU(3)_c \equiv SU(3)_{L+R}}, {\rm U(1)}_Y \equiv {\rm U(1)_{L+R}}$ and $Y = Y_{\rm L} = Y_{\rm R}$. Our interest is in the region $\Lambda_{\rm L} < \mu \leq \Lambda_{\rm X5}$. Hereafter, we call the range $\Lambda_{\rm L} < \mu \leq \Lambda_{\rm R}$, $\Lambda_{\rm R} < \mu \leq \Lambda_{\rm S}$ and $\Lambda_{\rm S} < \mu \leq \Lambda_{\rm X5}$ range I, II and III, respectively.

The electric charge operator Q is given by

$$Q = I_3^{\rm L} + \frac{1}{2} Y' \quad \text{(Range I)}, \tag{3.2}$$

$$\frac{1}{2} Y' = I_3^{\rm R} + \frac{1}{2} Y \quad \text{(Range II)}, \tag{3.3}$$

$$\frac{1}{2}Y = \frac{1}{2}Y_{\rm L} + \frac{1}{2}Y_{\rm R} \quad \text{(Range III)}. \tag{3.4}$$

We denote the gauge coupling constants corresponding to the operators Q, Y', Y, $Y_{\rm L}$, $Y_{\rm R}$, $I^{\rm L}$ and $I^{\rm R}$ by $g_{\rm em} \equiv e$, g_1', g_1, $g_{1\rm L}$, $g_{1\rm R}$, $g_{2\rm L}$ and $g_{2\rm R}$, respectively. The boundary conditions for these gauge coupling constants at $\mu = \Lambda_{\rm L}$, $\mu = \Lambda_{\rm R}$ and $\mu = \Lambda_{\rm S}$ are as follows:

$$\alpha_{\rm em}^{-1}(\Lambda_{\rm L}) = \alpha_{2\rm L}^{-1}(\Lambda_{\rm L}) + \frac{5}{3}\alpha_1'^{-1}(\Lambda_{\rm L}), \tag{3.5}$$

$$\frac{5}{3}\alpha_1'^{-1}(\Lambda_{\rm R}) = \alpha_{2\rm R}^{-1}(\Lambda_{\rm R}) + \frac{2}{3}\alpha_1^{-1}(\Lambda_{\rm R}), \tag{3.6}$$

and

$$\frac{2}{3}\alpha_1^{-1}(\Lambda_{\rm S}) = \frac{5}{3}\alpha_{1\rm L}^{-1}(\Lambda_{\rm S}) + \frac{5}{3}\alpha_{1\rm R}^{-1}(\Lambda_{\rm S}), \tag{3.7}$$

respectively, corresponding to (3.2), (3.3) and (3.4), where $\alpha_i \equiv g_i^2/4\pi$ and the normalizations of the $\mathrm{U}(1)_{Y'}$, $\mathrm{U}(1)_Y$, $\mathrm{U}(1)_{Y_{\rm L}}$ and $\mathrm{U}(1)_{Y_{\rm R}}$ gauge coupling constants have been taken as they satisfy $\alpha_1' = \alpha_{2\rm L} = \alpha_3$, $\alpha_{1\rm L} = \alpha_{2\rm L} = \alpha_{3\rm L}$ and $\alpha_{1\rm R} = \alpha_{2\rm R} = \alpha_{3\rm R}$ in the SU(5) grand-unification limit and $\alpha_1 = \alpha_3 \equiv \alpha_4$ in the SU(4) unification limit [$\alpha_4 = \alpha_{2\rm L} = \alpha_{2\rm R}$ in the SO(10) unification limit], respectively. We also have the following boundary conditions at $\mu = \Lambda_{\rm S}$ and $\mu = \Lambda_{X5}$:

$$\alpha_3^{-1}(\Lambda_{\rm S}) = \alpha_{3\rm L}^{-1}(\Lambda_{\rm S}) + \alpha_{3\rm R}^{-1}(\Lambda_{\rm S}), \tag{3.8}$$

$$\alpha_{1\rm L}^{-1}(\Lambda_{\rm X5L}) = \alpha_{2\rm L}^{-1}(\Lambda_{\rm X5L}) = \alpha_{3\rm L}^{-1}(\Lambda_{\rm X5L}), \tag{3.9}$$

$$\alpha_{1\rm R}^{-1}(\Lambda_{\rm X5R}) = \alpha_{2\rm R}^{-1}(\Lambda_{\rm X5R}) = \alpha_{3\rm R}^{-1}(\Lambda_{\rm X5R}), \tag{3.10}$$

where, for convenience, we distinguish the unification scale of $\mathrm{SU}(5)_{\rm L}$, Λ_{X5L}, from that of $\mathrm{SU}(5)_{\rm R}$, $\Lambda_{\rm X5R}$.

The evolutions of the gauge coupling constants g_i at one loop are given by the equations

$$\frac{\rm d}{{\rm d}t}\alpha_i(\mu) = -\frac{1}{2\pi}b_i\alpha_i^2(\mu)\ , \tag{3.11}$$

where $t = \ln\mu$. Since the quantum numbers of the fermions f and F are assigned as in Table 1, the coefficients b_i are as given in Table 2. In the model with $\det M_U = 0$, the heavy fermions $F_{\rm L}$ and $F_{\rm R}$ except for $U_{3\rm L}$ and $U_{3\rm R}$ are decoupled for $\mu \leq \Lambda_{\rm S}$, and the fermions $u_{3\rm R}$ and $U_{3\rm L}$ are decoupled for $\mu \leq \Lambda_{\rm R}$. In Table 2, we have also shown the values of b_i for the conventional case without the constraint $\det M_U = 0$ in parentheses.

By substituting $\alpha_{2\rm L}^{-1}(\Lambda_{\rm X5L}) = \alpha_{3\rm L}^{-1}(\Lambda_{\rm X5L})$ with the relations at one loop

$$\alpha_{2\rm L}^{-1}(\Lambda_{\rm X5L}) = \alpha_{2\rm L}^{-1}(\Lambda_S) + b_{2\rm L}^{\rm III}\frac{1}{2\pi}\ln\frac{\Lambda_{\rm X5L}}{\Lambda_{\rm S}}, \tag{3.12}$$

$$\alpha_{3\rm L}^{-1}(\Lambda_{\rm X5L}) = \alpha_{3\rm L}^{-1}(\Lambda_S) + b_{3\rm L}^{\rm III}\frac{1}{2\pi}\ln\frac{\Lambda_{\rm X5L}}{\Lambda_{\rm S}}, \tag{3.13}$$

we obtain

$$\alpha_{3\rm L}^{-1}(\Lambda_S) - \alpha_{2\rm L}^{-1}(\Lambda_S) + (b_{3\rm L}^{\rm III} - b_{2\rm L}^{\rm III})\frac{1}{2\pi}\ln\frac{\Lambda_{\rm X5L}}{\Lambda_{\rm S}} = 0. \tag{3.14}$$

Table 1. Quantum numbers of the fermions f and F and Higgs scalars $\phi_{\rm L}$, $\phi_{\rm R}$ and Φ for $\mathrm{SU}(2)_{\rm L} \times \mathrm{SU}(2)_{\rm R} \times \mathrm{U}(1)_Y$

	$I_3^{\rm L}$	$I_3^{\rm R}$	Y		$I_3^{\rm L}$	$I_3^{\rm R}$	Y
$u_{\rm L}$	$+\frac{1}{2}$	0	$\frac{1}{3}$	$u_{\rm R}$	0	$+\frac{1}{2}$	$\frac{1}{3}$
$d_{\rm L}$	$-\frac{1}{2}$	0	$\frac{1}{3}$	$d_{\rm R}$	0	$-\frac{1}{2}$	$\frac{1}{3}$
$\nu_{\rm L}$	$+\frac{1}{2}$	0	-1	$\nu_{\rm R}$	0	$+\frac{1}{2}$	-1
$e_{\rm L}$	$-\frac{1}{2}$	0	-1	$e_{\rm R}$	0	$-\frac{1}{2}$	-1
$U_{\rm L}$	0	0	$\frac{4}{3}$	$U_{\rm R}$	0	0	$\frac{4}{3}$
$D_{\rm L}$	0	0	$-\frac{2}{3}$	$D_{\rm R}$	0	0	$-\frac{2}{3}$
$N_{\rm L}$	0	0	0	$N_{\rm R}$	0	0	0
$E_{\rm L}$	0	0	-2	$E_{\rm R}$	0	0	-2
$\phi_{\rm L}^+$	$+\frac{1}{2}$	0	1	$\phi_{\rm R}^+$	0	$+\frac{1}{2}$	1
$\phi_{\rm L}^0$	$-\frac{1}{2}$	0	1	$\phi_{\rm R}^0$	0	$-\frac{1}{2}$	1

Similarly, from the condition $\alpha_{1\rm L}^{-1}(\Lambda_{\rm X5L}) = \alpha_{2\rm L}^{-1}(\Lambda_{\rm X5L})$ we obtain

$$\alpha_{2\rm L}^{-1}(\Lambda_S) - \alpha_{1\rm L}^{-1}(\Lambda_S) + (b_{2\rm L}^{\rm III} - b_{1\rm L}^{\rm III})\frac{1}{2\pi}\ln\frac{\Lambda_{\rm X5L}}{\Lambda_{\rm S}} = 0. \tag{3.15}$$

By eliminating $\ln(\Lambda_{\rm X5L}/\Lambda_{\rm S})$ from (3.14) and (3.15), we obtain

$$(b_{2\rm L}^{\rm III} - b_{1\rm L}^{\rm III})\alpha_{3\rm L}^{-1}(\Lambda_S) + (b_{3\rm L}^{\rm III} - b_{2\rm L}^{\rm III})\alpha_{1\rm L}^{-1}(\Lambda_S) - (b_{3\rm L}^{\rm III} - b_{1\rm L}^{\rm III})\alpha_{2\rm L}^{-1}(\Lambda_S) = 0. \tag{3.16}$$

Similarly, we obtain

$$(b_{2\rm R}^{\rm III} - b_{1\rm R}^{\rm III})\alpha_{3\rm R}^{-1}(\Lambda_S) + (b_{3\rm R}^{\rm III} - b_{2\rm R}^{\rm III})\alpha_{1\rm R}^{-1}(\Lambda_S) - (b_{3\rm R}^{\rm III} - b_{1\rm R}^{\rm III})\alpha_{2\rm R}^{-1}(\Lambda_S) = 0. \tag{3.17}$$

Therefore, from the relations (3.7), (3.8) and $b_{i\rm L}^{\rm III} = b_{i\rm R}^{\rm III} \equiv b_i^{\rm III}$, we obtain

$$(b_2^{\rm III} - b_1^{\rm III})\alpha_3^{-1}(\Lambda_S) + (b_3^{\rm III} - b_2^{\rm III})\alpha_1^{-1}(\Lambda_S) - (b_3^{\rm III} - b_1^{\rm III})\left[\alpha_{2\rm L}^{-1}(\Lambda_S) + \alpha_{2\rm R}^{-1}(\Lambda_S)\right] = 0, \tag{3.18}$$

which leads to

$$\begin{aligned}
&\left[\frac{3}{5}(b_3^{\rm III} - b_2^{\rm III}) + (b_3^{\rm III} - b_1^{\rm III})\right]\alpha_{2\rm R}^{-1}(\Lambda_{\rm R}) \\
&-\Big[b_3^{\rm II}(b_2^{\rm III} - b_1^{\rm III}) + \frac{2}{5}b_1^{\rm II}(b_3^{\rm III} - b_2^{\rm III}) - 2b_2^{\rm II}(b_3^{\rm III} - b_1^{\rm III})\Big]\frac{1}{2\pi}\ln\frac{\Lambda_S}{\Lambda_{\rm R}} \\
&-\big[b_3^{\rm I}(b_2^{\rm III} - b_1^{\rm III}) + b_1^{\rm I}(b_3^{\rm III} - b_2^{\rm III}) - b_2^{\rm I}(b_3^{\rm III} - b_1^{\rm III})\big]\frac{1}{2\pi}\ln\frac{\Lambda_{\rm R}}{\Lambda_{\rm L}} \\
&= (b_2^{\rm III} - b_1^{\rm III})\alpha_3^{-1}(\Lambda_{\rm L}) + (b_3^{\rm III} - b_2^{\rm III})\alpha_1'^{-1}(\Lambda_{\rm L}) - (b_3^{\rm III} - b_1^{\rm III})\alpha_{2\rm L}^{-1}(\Lambda_{\rm L}).
\end{aligned} \tag{3.19}$$

Table 2. Coefficients in the evolution equations of the gauge coupling constants. Cases (A) and (B) are the cases with the symmetry-breaking patterns SO(10) → SU(5) × U(1) and SO(10) → SU(2) × SU(2) × SU(4); they are discussed in Sects. 3 and 4, respectively

	$\Lambda_L < \mu \le \Lambda_R$	$\Lambda_R < \mu \le \Lambda_S$	$\Lambda_S < \mu \le \Lambda_X$ Case A	$\Lambda_S < \mu \le \Lambda_X$ Case B
$SU(3)_c$	$b_3^I = 7$	$b_3^{II} = 19/3$ (7)	$b_{3L}^{III} = 6$, $b_{3R}^{III} = 6$	$b_{4L}^{III} = 7$, $b_{4R}^{III} = 7$
$SU(2)_L$	$b_{2L}^I = 19/6$	$b_{2L}^{II} = 19/6$ (19/6)	$b_{2L}^{III} = 19/6$	$b_{2L}^{III} = 19/6$
$SU(2)_R$	$b_1^I = -41/10$	$b_{2R}^{II} = 19/6$ (19/6)	$b_{2R}^{III} = 19/6$	$b_{2R}^{III} = 19/6$
$U(1)_Y$		$b_1^{II} = -43/6$ $(-9/2)$	$b_{1L}^{III} = -53/10$, $b_{1R}^{III} = -53/10$	$b'_{2L} = -13/6$, $b'_{2R} = -13/6$

For the model with $\det M_U = 0$, the relation (3.19) becomes

$$13\alpha_{2R}^{-1}(\Lambda_R) + \frac{391}{15}\frac{1}{2\pi}\ln\frac{\Lambda_S}{\Lambda_R} - \frac{178}{15}\frac{1}{2\pi}\ln\frac{\Lambda_R}{\Lambda_L}$$
$$= \frac{127}{15}\alpha_3^{-1}(\Lambda_L) + \frac{17}{6}\alpha_1'^{-1}(\Lambda_L) - \frac{113}{30}\alpha_{2L}^{-1}(\Lambda_L). \quad (3.20)$$

The right-hand side of (3.20) gives the value −97.82 for the input values $\alpha'_1(m_Z) = 0.01683$, $\alpha_L(m_Z) = 0.03349$ and $\alpha_3(m_Z) = 0.1189$ [14], where, for convenience, we have used the initial values at $\mu = m_Z$ instead of those at $\mu = \Lambda_L$. The relation (3.20) puts a lower bound on the ratio Λ_R/Λ_L: For $\alpha_{2R}^{-1}(\Lambda_R) \ge 1$, we obtain $\Lambda_R/\Lambda_L \ge 2\times 10^{135}$ (for $\Lambda_S/\Lambda_R = 50$ [1]) and $\Lambda_R/\Lambda_L \ge 3\times 10^{22}$ (for $\Lambda_S/\Lambda_R \ge 1$). Such a large value of Λ_R/Λ_L is physically unlikely, so that case (A) is ruled out.

By a discussion similar to that of relation (3.19), it turns out that the conclusion that case (A) is ruled out is still unchanged for the model without the condition $\det M_U = 0$ and also for the minimal SUSY version of the present model.

4 Case of SO(10) → SU(2) × SU(2) × SU(4)

Next, we investigate case (B), $SO(10)_L \times SO(10)_R \to [SU(2) \times SU(2)' \times SU(4)]_L \times [SU(2) \times SU(2)' \times SU(4)]_R$. At the energy scale $\mu = \Lambda_S$, the symmetries $[SU(2)' \times SU(4)]_L \times [SU(2)' \times SU(4)]_R$ are broken into $U(1)_Y \times SU(3)_c$ by the Higgs scalars

$$\begin{aligned}\Phi_V &\sim (1,2,4;1,2,4),\\ \Phi_L &\sim (1,1,10;1,1,1),\\ \Phi_R &\sim (1,1,1;1,1,10),\end{aligned} \quad (4.1)$$

of $[SU(2)\times SU(2)'\times SU(4)]_L \times [SU(2)\times SU(2)'\times SU(4)]_R$, where the Higgs scalars Φ_V, Φ_L and Φ_R generate the masses M_F, M_L and M_R, respectively. In the present section, we call the ranges $\Lambda_L < \mu \le \Lambda_R$, $\Lambda_R < \mu \le \Lambda_S$ and $\Lambda_S < \mu \le \Lambda_X$ ranges I, II and III, respectively.

The electric-charge operator Q is given by (3.2) and (3.3) in the ranges I and II, respectively, but the relation (3.4) is replaced by

$$\frac{1}{2}Y = I_3'^L + \frac{1}{2}Y_L + I_3'^R + \frac{1}{2}Y_R, \quad (4.2)$$

so that the boundary condition (3.7) is replaced by

$$\frac{2}{3}\alpha_1^{-1}(\Lambda_S) = \alpha_{2L}'^{-1}(\Lambda_S) + \frac{2}{3}\alpha_{1L}^{-1}(\Lambda_S) + \alpha_{2R}'^{-1}(\Lambda_S) + \frac{2}{3}\alpha_{1R}^{-1}(\Lambda_S). \quad (4.3)$$

The boundary conditions at $\mu = \Lambda_S$ and $\mu = \Lambda_X$ are as follows:

$$\alpha_3^{-1}(\Lambda_S) = \alpha_{3L}^{-1}(\Lambda_S) + \alpha_{3R}^{-1}(\Lambda_S), \quad (4.4)$$
$$\alpha_{1L}^{-1}(\Lambda_S) = \alpha_{3L}^{-1}(\Lambda_S) = \alpha_{4L}^{-1}(\Lambda_S), \quad (4.5)$$
$$\alpha_{1R}^{-1}(\Lambda_S) = \alpha_{3R}^{-1}(\Lambda_S) = \alpha_{4R}^{-1}(\Lambda_S), \quad (4.6)$$
$$\alpha_{2L}^{-1}(\Lambda_{XL}) = \alpha_{2L}'^{-1}(\Lambda_{XL}) = \alpha_{4L}^{-1}(\Lambda_{XL}), \quad (4.7)$$
$$\alpha_{2R}^{-1}(\Lambda_{XR}) = \alpha_{2R}'^{-1}(\Lambda_{XR}) = \alpha_{4R}^{-1}(\Lambda_{XR}), \quad (4.8)$$

where, for convenience, we have again distinguished the unification scale of $SO(10)_L$, Λ_{XL}, from that of $SO(10)_R$, Λ_{XR}.

Since $b_{2L}'^{III} = b_{2R}'^{III} \equiv b_2'^{III} \ne b_{2L}^{III} = b_{2R}^{III} \equiv b_2^{III}$, we obtain

$$\alpha_{2L}'^{-1}(\Lambda_S) - \alpha_{2L}^{-1}(\Lambda_S) = (b_{2L}'^{III} - b_{2L}^{III})\frac{1}{2\pi}\ln\frac{\Lambda_S}{\Lambda_{XL}}, \quad (4.9)$$

$$\alpha_{2R}'^{-1}(\Lambda_S) = \alpha_{2R}^{-1}(\Lambda_S) = (b_{2R}'^{III} - b_{2R}^{III})\frac{1}{2\pi}\ln\frac{\Lambda_S}{\Lambda_{XR}}, \quad (4.10)$$

i.e.,

$$\alpha_{2L}'^{-1}(\Lambda_S) + \alpha_{2R}'^{-1}(\Lambda_S)$$
$$= \alpha_{2L}^{-1}(\Lambda_S) + \alpha_{2R}^{-1}(\Lambda_S) + 2(b_2^{III} - b_2'^{III})\frac{1}{2\pi}\ln\frac{\Lambda_X}{\Lambda_S}, \quad (4.11)$$

where $\Lambda_X = (\Lambda_{XL}\Lambda_{XR})^{1/2}$. On the other hand, from (4.3)–(4.6), we obtain

$$\alpha_3^{-1}(\Lambda_S) + \frac{3}{2}\left[\alpha_{2L}'^{-1}(\Lambda_S) + \alpha_{2R}'^{-1}(\Lambda_S)\right] - \alpha_1^{-1}(\Lambda_S) = 0, \quad (4.12)$$

so that

$$\alpha_3^{-1}(\Lambda_S) + \frac{3}{2}\left[\alpha_{2L}^{-1}(\Lambda_S) + \alpha_{2R}^{-1}(\Lambda_S)\right]$$
$$-\alpha_1^{-1}(\Lambda_S) + 3(b_2^{III} - b_2^{\prime III})\frac{1}{2\pi}\ln\frac{\Lambda_X}{\Lambda_S} = 0. \quad (4.13)$$

Similarly, from (4.7), we obtain

$$\alpha_{3L}^{-1}(\Lambda_S) - \alpha_{2L}^{-1}(\Lambda_S) + (b_{4L}^{III} - b_{2L}^{III})\frac{1}{2\pi}\ln\frac{\Lambda_{XL}}{\Lambda_S} = 0, \quad (4.14)$$

so that, together with the equation with (L → R) in (4.14), we obtain

$$\alpha_3^{-1}(\Lambda_S) - \left[\alpha_{2L}^{-1}(\Lambda_S) + \alpha_{2R}^{-1}(\Lambda_S)\right]$$
$$+2(b_4^{III} - b_2^{III})\frac{1}{2\pi}\ln\frac{\Lambda_X}{\Lambda_S} = 0. \quad (4.15)$$

By eliminating Λ_X/Λ_R from (4.13) and (4.15), we obtain

$$c_3\ \alpha_3^{-1}(\Lambda_S) + c_2\left[\alpha_{2L}^{-1}(\Lambda_S) + \alpha_{2R}^{-1}(\Lambda_S)\right] - c_1\ \alpha_1^{-1}(\Lambda_S) = 0, \quad (4.16)$$

where

$$c_1 = b_4^{III} - b_2^{III}, \quad (4.17)$$

$$c_2 = \frac{3}{2}(b_4^{III} - b_2^{\prime III}), \quad (4.18)$$

$$c_3 = b_4^{III} - b_2^{III} - \frac{3}{2}(b_2^{III} - b_2^{\prime III}). \quad (4.19)$$

Since

$$\alpha_1^{-1}(\Lambda_S) = \frac{5}{2}\alpha_1^{-1}(\Lambda_L) - \frac{3}{2}\alpha_{2R}^{-1}(\Lambda_R)$$
$$+ b_1^{II}\frac{1}{2\pi}\ln\frac{\Lambda_S}{\Lambda_R} + \frac{5}{2}b_1^{II}\frac{1}{2\pi}\ln\frac{\Lambda_R}{\Lambda_L}, \quad (4.20)$$

$$\alpha_{2L}^{-1}(\Lambda_S) = \alpha_{2L}^{-1}(\Lambda_L) + b_2^{II}\frac{1}{2\pi}\ln\frac{\Lambda_S}{\Lambda_R} + b_2^{I}\frac{1}{2\pi}\ln\frac{\Lambda_R}{\Lambda_L}, \quad (4.21)$$

$$\alpha_{2R}^{-1}(\Lambda_S) = \alpha_{2L}^{-1}(\Lambda_R) + b_2^{II}\frac{1}{2\pi}\ln\frac{\Lambda_S}{\Lambda_R}, \quad (4.22)$$

$$\alpha_3^{-1}(\Lambda_S) = \alpha_3^{-1}(\Lambda_L) + b_3^{II}\frac{1}{2\pi}\ln\frac{\Lambda_S}{\Lambda_R} + b_3^{I}\frac{1}{2\pi}\ln\frac{\Lambda_R}{\Lambda_L}, \quad (4.23)$$

the relation (4.16) leads to the constraint for Λ_R/Λ_L:

$$0 = \left(c_2 + \frac{3}{2}c_1\right)\alpha_{2R}^{-1}(\Lambda_R)$$
$$+ (c_3 b_3^{II} + 2c_2 b_2^{II} - c_1 b_1^{II})\frac{1}{2\pi}\ln\frac{\Lambda_S}{\Lambda_R}$$
$$+ \left(c_3 b_3^{I} + c_2 b_2^{I} - \frac{5}{2}b_1^{I}\right)\frac{1}{2\pi}\ln\frac{\Lambda_R}{\Lambda_L}$$
$$+ c_3\alpha_3^{-1}(\Lambda_L) + c_2\alpha_{2L}^{-1}(\Lambda_L) - \frac{5}{2}\alpha_1^{-1}(\Lambda_L)$$
$$= 19.5\,\alpha_{2R}^{-1}(\Lambda_R) + 19.67\log\frac{\Lambda_R}{\Lambda_L}$$
$$+ 32.31\log\frac{\Lambda_S}{\Lambda_R} - 193.96, \quad (4.24)$$

Table 3. Intermediate mass scales Λ_R and Λ_S versus $\alpha_{2R}^{-1}(\Lambda_R)$ in the case of SO(10) → SU(2)×SU(2)×SU(4). As input values $\Lambda_R/\Lambda_S = 0.02$ and $\Lambda_L = m_Z = 91.2$ GeV are used. The upper and lower rows of Λ_{XR} and Λ_{XL} correspond to the values of $\alpha_{4R}^{-1}(\Lambda_S) = 1$ and $\alpha_{4R}^{-1}(\Lambda_S) = 2$, respectively

$\alpha_{2R}^{-1}(\Lambda_R)$	1	2	4	6
Λ_R/Λ_L	1.20×10^6	1.23×10^5	1.27×10^3	1.32×10^1
Λ_R [GeV]	1.10×10^8	1.12×10^7	1.16×10^5	1.21×10^3
Λ_S [GeV]	5.48×10^9	5.59×10^8	5.81×10^6	6.04×10^4
Λ_X [GeV]	4.88×10^{14}	3.53×10^{13}	1.86×10^{14}	9.75×10^{13}
Λ_{XR} [GeV]	1.39×10^{11}	7.29×10^{10}	2.01×10^{10}	5.54×10^9
	2.69×10^{10}	1.41×10^{10}	3.90×10^9	1.08×10^9
Λ_{XL} [GeV]	1.71×10^{18}	1.71×10^{18}	1.71×10^{18}	1.71×10^{18}
	8.83×10^{18}	8.83×10^{18}	8.83×10^{18}	8.83×10^{18}

where we have used the values of b_i given in Table 2 and the same input values of $\alpha_1^{-1}(\Lambda_L)$, $\alpha_2^{-1}(\Lambda_L)$ and $\alpha_3^{-1}(\Lambda_L)$ as used in (3.20). For $\Lambda_S/\Lambda_R = 50$, the relation (4.24) leads to

$$\log\frac{\Lambda_R}{\Lambda_L} = 7.071 - 0.9915\,\alpha_{2R}^{-1}(\Lambda_R), \quad (4.25)$$

so that, for $\alpha_{2R}^{-1}(\Lambda_R) \geq 1$, we obtain the constraint

$$\kappa \equiv \Lambda_R/\Lambda_L \leq 1.20\times10^6. \quad (4.26)$$

Similarly, we can obtain the constraint for Λ_X/Λ_S:

$$\log\frac{\Lambda_X}{\Lambda_S} = 4.098 + 0.8517\,\alpha_{2R}^{-1}(\Lambda_R). \quad (4.27)$$

We show the values of Λ_R, Λ_S and Λ_X for the typical values of $\alpha_{2R}^{-1}(\Lambda_R)$ in Table 3. The values of Λ_{XL} and Λ_{XR} depend not only on the input value of $\alpha_{2R}^{-1}(\Lambda_R)$ but also on that of $\alpha_{4R}^{-1}(\Lambda_S)$, because

$$\alpha_{4R}^{-1}(\Lambda_S) = \alpha_{2R}^{-1}(\Lambda_S) + (b_2^{III} - b_4^{III})\frac{1}{2\pi}\ln\frac{\Lambda_{XR}}{\Lambda_S}$$
$$= \frac{1}{2}\left[\alpha_{2R}^{-1}(\Lambda_S) - \alpha_{2L}^{-1}(\Lambda_S) + \alpha_3^{-1}(\Lambda_S)\right]$$
$$+ (b_4^{III} - b_4^{III})\frac{1}{2\pi}\ln\frac{\Lambda_X}{\Lambda_{XR}} \quad (4.28)$$
$$= -3.785 - 0.1964\,\alpha_{2R}^{-1}(\Lambda_R) + 1.405\log\frac{\Lambda_X}{\Lambda_{XR}},$$

i.e.,

$$\log\frac{\Lambda_X}{\Lambda_{XR}} = 2.694 + 0.1398\,\alpha_{2R}^{-1}(\Lambda_R) + 0.7118\,\alpha_{4R}^{-1}(\Lambda_S), \quad (4.29)$$

where we have used $\Lambda_S/\Lambda_R = 50$. For $\alpha_{2R}^{-1}(\Lambda_R) \geq 1$ and $\alpha_{4R}^{-1}(\Lambda_S) \geq 1$, the relation (4.29) gives the constraint

$$\Lambda_{XL}/\Lambda_{XR} \geq 1.26\times10^7. \quad (4.30)$$

The relation (4.29) leads us to conclude that a model with $\Lambda_{XL} = \Lambda_{XR}$ is ruled out. The values of Λ_{XR} and Λ_{XL} for

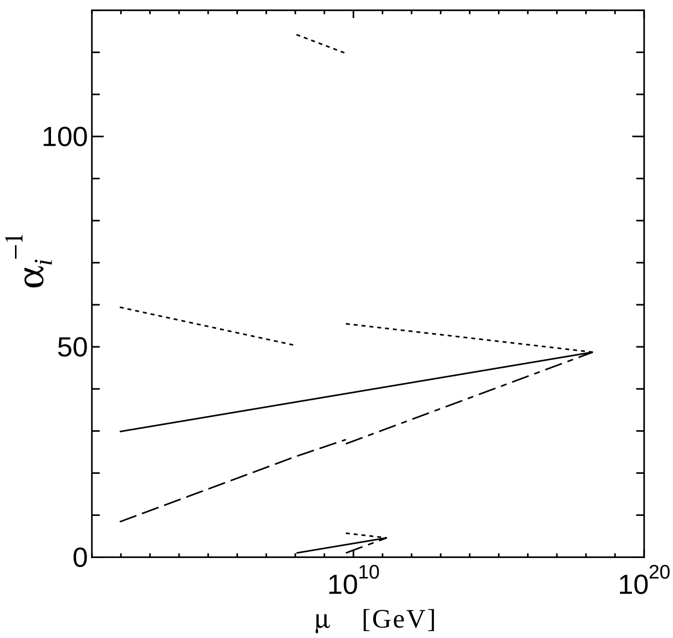

Fig. 1. Behaviors of $\alpha_1'^{-1}(\mu)$ (dotted line) with $\Lambda_L < \mu \le \Lambda_R$, $\alpha_1^{-1}(\mu)$ (dotted line) with $\Lambda_R < \mu \le \Lambda_S$, $\alpha_{2L}^{-1}(\mu)$ (solid line) with $\Lambda_L < \mu \le \Lambda_{XL}$, $\alpha_{2R}^{-1}(\mu)$ (solid line) with $\Lambda_R < \mu \le \Lambda_{XR}$, $\alpha_3^{-1}(\mu)$ (dashed line) with $\Lambda_L < \mu \le \Lambda_S$, $\alpha_{2L}'^{-1}(\mu)$ (dotted line) with $\Lambda_S < \mu \le \Lambda_{XL}$, and $\alpha_{2R}'^{-1}(\mu)$ (dotted line) with $\Lambda_S < \mu \le \Lambda_{XR}$, $\alpha_{4L}^{-1}(\mu)$ (dotted chain line) with $\Lambda_S < \mu \le \Lambda_{XL}$, and $\alpha_{4R}^{-1}(\mu)$ (dotted chain line) with $\Lambda_S < \mu \le \Lambda_{XR}$, where $\Lambda_L = 91.2\,\mathrm{GeV}$, $\Lambda_R = 1.10 \times 10^8\,\mathrm{GeV}$, $\Lambda_S = 5.48 \times 10^9\,\mathrm{GeV}$, $\Lambda_{XR} = 1.39 \times 10^{11}\,\mathrm{GeV}$ and $\Lambda_{XL} = 1.71 \times 10^{18}\,\mathrm{GeV}$. The values $\alpha_1'^{-1}(\Lambda_L) = 59.42$, $\alpha_{2L}^{-1}(\Lambda_L) = 29.86$, $\alpha_3^{-1}(\Lambda_L) = 8.410$, $\alpha_{2R}^{-1}(\Lambda_R) = 1$ and $\alpha_{4R}^{-1}(\Lambda_S) = 1$ are used as the input values

typical values of $\alpha_{2R}^{-1}(\Lambda_R)$ and $\alpha_{4R}^{-1}(\Lambda_S)$ are also listed in Table 3.

Considering the present results [14] of the experimental search for the right-handed weak bosons, we take $\kappa \equiv \Lambda_R/\Lambda_L \ge 10$, so that we conclude that the allowed ranges of κ, the intermediate energy scale Λ_S and the unification scale $\Lambda_X \equiv (\Lambda_{XL}\Lambda_{XL})^{1/2}$ are

$$\kappa = 1.3 \times 10^1\text{–}1.2 \times 10^6,$$
$$\Lambda_S = (6.0 \times 10^4\text{–}5.5 \times 10^9)\,\mathrm{GeV}, \quad (4.31)$$
$$\Lambda_X = (9.8 \times 10^{13}\text{–}4.9 \times 10^{14})\,\mathrm{GeV},$$

corresponding to the values $\alpha_{2R}^{-1}(\Lambda_R) = 6$–1. The behavior of the gauge coupling constants for a typical case is illustrated in Fig. 1.

5 Conclusions

In conclusion, in order to examine the idea that the extra fermions F_R and F_L in the universal seesaw mass-matrix model, together with the conventional three families of quarks and leptons f_L and f_R, are assigned to $(f_L + F_R^c) \sim (16,1)$ and $(f_R + F_L^c) \sim (1,16)$ of $\mathrm{SO}(10)_L \times \mathrm{SO}(10)_R$, we have investigated the evolution of the gauge coupling constants and intermediate mass scales. Case (A), $\mathrm{SO}(10)_L \times \mathrm{SO}(10)_R \to [\mathrm{SU}(5) \times \mathrm{U}(1)']_L \times [\mathrm{SU}(5) \times \mathrm{U}(1)']_R$, is ruled out because the results are inconsistent with the observed values of the gauge coupling constants at $\mu = m_Z$. Case (B), $\mathrm{SO}(10)_L \times \mathrm{SO}(10)_R \to [\mathrm{SU}(2) \times \mathrm{SU}(2)' \times \mathrm{SU}(4)]_L \times [\mathrm{SU}(2) \times \mathrm{SU}(2)' \times \mathrm{SU}(4)]_R$, is allowed for the intermediate energy scale $\Lambda_R \sim (10^1 - 10^6)\,\mathrm{GeV}$ if we accept a model with $\Lambda_{XL} \neq \Lambda_{XR}$, where Λ_{XL} and Λ_{XR} are the unification scales of $\mathrm{SO}(10)_L$ and $\mathrm{SO}(10)_R$, respectively. We have obtained the allowed ranges $\kappa \simeq 10^1$–10^6, $\Lambda_S \simeq (6\times10^4$–$6\times10^9)\,\mathrm{GeV}$, and $\Lambda_X = (\Lambda_{XL}\Lambda_{XR})^{1/2} = (5\times10^{14}$–$10^{14})\,\mathrm{GeV}$ corresponding to $\alpha_{2R}^{-1}(\Lambda_R) \simeq 6$–1.

In case (B), since $M_L \sim M_R \sim M_N \sim M_F$ $(F \neq N)$, we see that this gives an effective neutrino-mass matrix $M(\nu_L) \simeq -m_L M_L^{-1} m_L^T$, so that the conventional neutrino masses $m(\nu_i)$ are of the order of $m(e_i)/\kappa$. However, for the condition $\alpha_{2R}^{-1}(\Lambda_R) \ge 1$, which is a condition leading to a perturbative model, the value of κ has been constrained by (4.26), i.e., $\kappa \le 1.20 \times 10^6$. This suggests that $m(\nu_\tau) \sim m(\tau)/\kappa \ge 10^3\,\mathrm{eV}$. Such a large value of $m(\nu_\tau)$ is unlikely. Therefore, the straightforward application of case (B) to the neutrino-mass generation scenario is ruled out.

However, the numerical results in Sect. 4 should not be taken rigidly, because the calculation was done at one loop. Moreover, the results are dependent on the input value Λ_R/Λ_S. The value $\Lambda_R/\Lambda_S = 0.02$ has been quoted from [1], where the value was determined from the observed value of m_c/m_t on the basis of a specific model for m_L, m_R and M_F. To be exact, the value 0.02 means $y_L v_L y_R v_R / y_S v_S = 0.02$, where the y's and v's are the Yukawa coupling constants and vacuum expectation values, respectively. Because of the numerical uncertainty of y_L, y_R and y_S, the numerical results may be changed by one or two orders. Case (B) still cannot be ruled out.

In the present paper, the cases of a SUSY version of the model have not been investigated systematically, because many versions for the energy scale of the SUSY partners of the super heavy fermions F can be considered. Nevertheless, case (A) can easily be ruled out by a simple consideration. On the other hand, for case (B), it is a future task to decide whether the SUSY version is allowed or not.

When we take the numerical result of the constraint (4.26), we can consider a minimum modification of case (B). In case (B), the Dirac mass matrix M_D is generated by the Higgs scalar $\Phi_V \sim (1,2,4;1,2,4)$ of $[\mathrm{SU}(2) \times \mathrm{SU}(2)' \times \mathrm{SU}(4)]_L \times [\mathrm{SU}(2) \times \mathrm{SU}(2)' \times \mathrm{SU}(4)]_R$, while the Majorana mass matrices M_L and M_R are generated by the Higgs scalars $\Phi_L \sim (1,1,10;1,1,1)$ and $\Phi_R \sim (1,1,1;1,1,10)$, respectively. We assume that the symmetries $\mathrm{SU}(4)_L$ and $\mathrm{SU}(4)_R$ are broken into $[\mathrm{SU}(3) \times \mathrm{U}(1)]_L$ and $[\mathrm{SU}(3) \times \mathrm{U}(1)]_R$ at $\mu = \Lambda_{NL} \equiv O(M_L)$ and $\mu = \Lambda_{NR} \equiv O(M_R)$, respectively, and that the energy scales Λ_{NL} and Λ_{NR} are sufficiently larger than $\Lambda_S \equiv O(M_D)$, at which all the fermions F (not f) have Dirac masses M_F and the symmetries $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ and $\mathrm{U}(1)_L \times \mathrm{U}(1)_R$ are broken into $\mathrm{SU}(3)_{L+R} \equiv \mathrm{SU}(3)_c$ and $\mathrm{U}(1)_{L+R} \equiv \mathrm{U}(1)_Y$, respectively.

Then, the neutrino-mass generation scenario is changed from scenario (b) to scenario (a). Although the expression of M_ν is still given by $M_\nu \simeq -m_L M_L^{-1} m_L^T$, the suppression factor for the neutrino masses is changed from $1/\kappa$ to $(1/\kappa)(\Lambda_S/\Lambda_{NL})$. By taking $\Lambda_S/\Lambda_{NL} \sim 10^{-3}$, we can obtain reasonable values for the neutrino masses in the case $\alpha_{2R}^{-1}(\Lambda_R) \simeq 1$. Of course, in the modified version with $\Lambda_{XL} \gg \Lambda_{NL} \gg \Lambda_S$, the unification scales of Λ_{XL} and Λ_{XR} are changed by an order of one or two. However, Λ_R and Λ_S are insensitive to the present modification.

In the present paper, we have not discussed the evolution of the Yukawa coupling constants. The phenomenological success in [1] has been obtained by taking $b_e = 0$, $b_u = -1/3$ and $b_d = -e^{i\beta_d}$ ($\beta_d = 18°$), where $M_F = m_0 \lambda_f \mathrm{diag}(1, 1, 1 + 3b_f)$ in the basis in which M_F is diagonal. The shapes (not the magnitudes) of $M_E = m_0\lambda_e \cdot\mathrm{diag}(1, 1, 1)$ and $M_U = m_0\lambda_u \mathrm{diag}(1, 1, 0)$ are almost invariant under the evolution, while the shape of $M_D \simeq m_0\lambda_d \mathrm{diag}(1, 1, -2)$ is not invariant. The following problems among others remain as our future tasks:
(i) What value of b_d is favorable at the unification scale $\mu = \Lambda_X$?
(ii) can we still assert $\lambda_u \simeq \lambda_d$ or not?
(iii) can the mass matrix m_R still be approximately diagonal in the basis in which m_L is diagonal? The numerical results in [1] will be somewhat changed in the present $\mathrm{SO(10)_L} \times \mathrm{SO(10)_R}$ model.

In any case, for the universal seesaw mass-matrix model based on the $\mathrm{SO(10)_L} \times SO(10)_R$ unification, if we consider the symmetry breaking $\mathrm{SO(10)_L} \times \mathrm{SO(10)_R} \to [\mathrm{SU(2)} \times \mathrm{SU(2)'} \times \mathrm{SU(4)}]_L \times [\mathrm{SU(2)} \times \mathrm{SU(2)'} \times \mathrm{SU(4)}]_R$, and we accept the case $\Lambda_{XL} \neq \Lambda_{XR}$, where Λ_{XL} and Λ_{XR} are the unification scales of $\mathrm{SO(10)_L}$ and $\mathrm{SO(10)_R}$, respectively, we can find a solution of the intermediate energy scales Λ_R and Λ_S for the unified description of the quark and lepton mass matrices, where only the top quark mass m_t is given by $m_t \sim \Lambda_L$ in contrast with $m_q \ll \Lambda_L$ ($q \neq t$), and the neutrino masses $m(\nu_i)$ are reasonably suppressed compared with the charged-lepton masses $m(e_i)$. The model is worth to be taken seriously as a promising unified model of the quarks and leptons.

Acknowledgements. The author would like to thank H. Fusaoka, T. Morozumi, M. Tanimoto and T. Matsuki for helpful discussions and for their useful comments. He also wishes to thank the Yukawa Institute of Theoretical Physics, Kyoto University, for its kind hospitality and for offering use of the computer facility. This work was supported by a Grant-in-Aid for Scientific Research, Ministry of Education, Science and Culture, Japan (No. 08640386).

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